bleqt
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bleqt is a c++ program with qt5 GUI for one dimensional two phase flow simulation in hydraulic fractures.

Note: This project is under active development.

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1.1 Task description

Let's consider the cylindrical part of reservoir with horizontal well and transverse hydraulic fracture inside. The well is cylindrical pipe with radius r_w and lenght 2L. The fracture is a circle with radius equals to reservoir radius. Both reservoir and fracture are saturated with fluid.

The geometry of reservoir and fracture as follows:

- R – reservoir radius, m;

- L – cylindr half height, m;

- r_w – well radius, m;

- δ – fracture half thickness, m.

Reservoir and fluid properties:

- k – reservoir permeability, m2;

- k_f – fracture permeability, m2;

- m_f – fracture porosity,

- μ – fluid viscosity, Pa*s.

The domain bounds are follows: Γ_B – side face of cylindrical reservoir, Γ_T – bearing faces of cylindrical reservoir, Γ_w – wellbore, Γ_f – fracture/reservoir bound, Γ_w^f – fracture/well bound.

Note: Fracture permeability and porosity are constant values.

The rate of hydraulically fractured horizontal well need to be defined.

1.2 Equations

Domain is a cylinder $D = [rw, R] * [0, 2\pi] * [-L, L]$, which includes fracture $D_f = [rw, R] * [0, 2\pi] * [-\delta, \delta]$.

Reservoir:

- **pressure:**

$$\text{div}(u) = 0, \quad u = -\sigma(\boldsymbol{x}, s) \cdot \nabla p, \quad \sigma = k(\boldsymbol{x})/\mu(s).$$

here $\boldsymbol{x}$ is a point, $\mu(s)-$ fluid viscosity, $u-$ flow rate, $s-$ (water) saturation, $k-$ - permeability, $\sigma-$ transmissibility of fluid.

Functions $\mu(s), f(s)$ defined from relative permeability degree functions

$$\frac{1}{\mu(s)} = \frac{\phi(s)}{\mu_w}, \quad \phi(s) = k_w(s) + k_\mu k_o(s), \quad k_\mu = \frac{\mu_w}{\mu_o},$$
$$k_w(s) = s^N, \quad k_o(s) = (1-s)^N.$$

here k_w- relative permeability of water, k_o- relative permeability of oil, μ_w- water viscosity, μ_o- oil viscosity.

Note: Note that irreducible water and oil saturations are equal to 0.

- **saturation:**

$$s = const.$$

Fracture: Let us define average value over pressure variables

$$\langle \cdot \rangle_f = \frac{1}{2\delta} \int_{-\delta}^{\delta} (\cdot) \, dz. \tag{1.1}$$

- **pressure:**

$$\text{div}(u_f) = 0, \quad u_f = -\sigma_f(s_f) \cdot \nabla p_f, \quad \sigma_f = k_f/\mu(s). \tag{1.2}$$

here u_f- flow rate in fracture, $s-$ (water) saturation, k_f- constant fracture permeability.

Assume that

$$< u_f >_r \approx -\frac{k_f}{\mu} \nabla < p_f > .$$

If we integrate equation (1.2) from $-\delta$ to δ using (1.1) and consider that we get (average signs $<>$ are gone)

$$\nabla_r u_f + \frac{1}{2\delta} \left. u_z \right|_{\Gamma_f(-\delta)}^{\Gamma_f(\delta)} = 0. \tag{1.3}$$

The last member in (1.3) defines the flow from reservoir to fracture from bounds $\Gamma_f(-\delta), \Gamma_f(\delta)$.

- **saturation:**

$$m_f \frac{\partial s_f}{\partial t} + \nabla_r \left(f(s_f) u_f \right) + \frac{1}{2\delta} \left. (f(s) u_z) \right|_{\Gamma_f(-\delta)}^{\Gamma_f(\delta)} = 0, \quad f(x) = \frac{k_w(x)}{\phi(x)}, \tag{1.4}$$

here $f(x)-$ Backley-Leverette function.

Conditions:

- **boundary:**

$$\Gamma_w^f \in [r_w, rw] * [0, 2\pi] * [-\delta, \delta] : p = p_w,$$
$$\Gamma_T \in [r_w, R] * [0, 2\pi] * [\pm L, \pm L] : p = p_c,$$
$$\Gamma_w \in [r_w, rw] * [0, 2\pi] * [-L, L]/[-\delta, \delta] : u_n = -\sigma \frac{\partial p}{\partial n} = 0,$$
$$\Gamma_B \in [R, R] * [0, 2\pi] * [-L, L] : u_n = 0,$$
$$\Gamma_f \in [R, R] * [0, 2\pi] * [\pm \delta, \pm \delta] : p = p_f, u = u_f, s = s_0.$$

- **initial:**

$$s = s_0 = const,$$
$$s_f = s_f^0.$$

Well flow rate:

- **from fract to well:**

$$q = \int_{\Gamma_w^f} u_f d\Gamma = 4\pi r_w \delta u_f.$$

- **from reservoir to fract:**

$$q = \int_{\Gamma_T} u d\Gamma.$$

1.3 Dimensionless

The following dimensionless parameters will be defined:

$$\bar{p} = \frac{p - p_c}{p_c - p_w},$$
$$\bar{r}, z = \frac{r, z}{R},$$
$$\bar{u} = \frac{u}{u_0}, u_0 = \frac{k}{\mu_w} \frac{p_c - p_w}{R},$$
$$\bar{u}_f = \frac{u_f}{u_f^0}, u_f^0 = \frac{k_f}{\mu_w} \frac{p_c - p_w}{R},$$
$$\bar{t} = \frac{t}{t_0}, t_0 = \frac{m_f R}{u_f^0},$$
$$\bar{q} = \frac{q}{q_0}, q_0 = R^2 u_f^0.$$

After applying dimensionless variables equation for pressure in reservoir will not change it's form.

Equations for pressure and saturation in fracture become (dimensionless sign is gone)

- **pressure:**

$$\nabla_r u_f + \frac{1}{2M} u_z|_{\Gamma_f(-\delta)}^{\Gamma_f(\delta)} = 0, \quad M = \frac{\delta k_f}{Rk}.$$

- **saturation:**

$$\frac{\partial s_f}{\partial t} + \nabla_r \left(f(s_f) u_f \right) + \frac{1}{2M} \left. \left(f(s) u_z \right) \right|_{\Gamma_f(-\delta)}^{\Gamma_f(\delta)} = 0.$$